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Supersymmetric defect expansion in CFT from AdS supertubes

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Abstract: The AdS/dCFT correspondence is used to show that a planar q-dimensional superconformal CFT defect expands, under the addition of electric charge and angular momentum, to a supersymmetric higher-dimensional defect of geometry $\mathbb{R}^q \times \mathcal{C}$, where $\mathcal{C}$ is an *arbitrary* curve. The dual string theory process is the expansion of D-branes and fundamental strings into a supertube in an AdS background.

Keywords: D-branes, Brane Dynamics in Gauge Theories, AdS/CFT and dS/CFT Correspondence, Supersymmetry and Duality.

Contents

1. Introduction

The AdS/dCFT correspondence [1] is an interesting extension of the AdS/CFT correspondence [2] in which additional structure is added on both sides. On the string theory side a brane that extends to the AdS boundary is added. On the CFT side we have a defect where the brane meets the AdS boundary. A necessary condition for the defect to preserve (part of) the conformal invariance of the bulk CFT is that the induced metric on the brane include an AdS factor; we will see, however, that this is not sufficient. In type-IIB string theory, an AdS/dCFT correspondence can be "derived" for each orthogonal q-dimensional intersection of N D3-branes and one Dp-brane, which we shall denote as $(q|\mathrm{D3} \perp \mathrm{D}p)$. For example, consider the 1/4-supersymmetric $(2|\mathrm{D3} \perp \mathrm{D5})$ intersection represented by the array

$$\begin{array}{lcccccccccc} \mathrm{D3:} & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ & \\ \mathrm{D5:} & \_ & 2 & 3 & 4 & 5 & 6 & \_ & \_ & \_ & . \end{array} \tag{1.1}$$

This is the case originally introduced in [1]. If $g_s \ll 1$ and $g_s N \gg 1$, where g_s is the string coupling constant, then the perturbative description of this system in the decoupling limit is in terms of an $AdS_4 \times S^2$ D5-brane probe in the $AdS_5 \times S^5$ near-horizon geometry of the D3-branes; the probe intersects the AdS boundary at an $\mathbb{R}^2$-defect. The AdS_4-factor arises from the 023-directions together with the radial coordinate in the 456-space, whereas the two angular coordinates in this space yield the S^2-factor. On the other hand, if $g_s N \ll 1$ then the perturbative description of the system is that of the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory in the presence of an $\mathbb{R}^2$-defect. The AdS/dCFT correspondence states that these two descriptions are actually

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equivalent throughout the entire parameter space. The degrees of freedom on the defect arise from the open strings connecting the D5-brane to the N D3-branes [3, 4], and are the holographic duals of the open-string degrees of freedom on the probe D5-brane [5].

More generally, every 1/4-supersymmetric $(q|\mathrm{D3} \perp \mathrm{D}p)$ intersection leads to an analogous duality involving a probe $AdS_{q+2} \times S^{p-q-2}$ Dp-brane in $AdS_5 \times S^5$ and an $\mathbb{R}^q$-defect in the SYM theory. Since each probe Dp-brane preserves eight Poincaré and eight special conformal supersymmetries [6] and (part of) the conformal symmetry in $AdS_5 \times S^5$, so does the dual defect in the gauge theory.

In the IIB Minkowski vacuum, the addition of angular momentum to a collection of orthogonal Dp-branes and fundamental strings causes them to expand to a D$(p+2)$-brane supertube [7], which we henceforth refer to as a D$(p+2)$-tube. This is a 1/4-supersymmetric tubular D$(p+2)$-brane of geometry $\mathbb{R}^{p+1} \times \mathcal{C}$, where the cross-section $\mathcal{C}$ is a completely *arbitrary* curve[1] in $\mathbb{R}^{8-p}$ [8]; the initial strings and Dp-branes appear on the D$(p+2)$-tube as dissolved charges represented by electric and magnetic Born-Infeld (BI) fields. Although originally found as a solution of the Dirac-Born-Infeld (DBI) action, the supertube is an exact solution (to all orders in α') of classical open-string theory [9]. One crucial feature of the D$(p+2)$-tube is that it preserves the same supersymmetries as a collection of strings and Dp-branes; in particular, there is no trace of a condition associated to the presence of the D$(p+2)$-brane [7]. This paper is based on the observation that the supersymmetric expansion to a D$(p+2)$-tube just described also takes place in the presence of (suitably oriented) D3-branes, in particular in their $AdS_5 \times S^5$ background. The holographically dual process in the CFT is a *supersymmetric defect expansion.*

Consider for concreteness the 1/8-supersymmetric intersection

$$\begin{array}{lccccccccc} \mathrm{D3}: & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ \\ \mathrm{D3}: & \_ & \_ & 3 & 4 & 5 & \_ & \_ & \_ & \_ \\ \mathrm{F1}: & \_ & \_ & \_ & \_ & \_ & 6 & \_ & \_ & \_ \;. \end{array} \tag{1.2}$$

This can be viewed as a $(1|\mathrm{D3} \perp \mathrm{D3})$ intersection to which fundamental strings have been added. If the D3-branes in the first line are replaced by an $AdS_5 \times S^5$ geometry then the D3-brane in the second line becomes an $AdS_3 \times S^1$ probe that intersects the boundary of $AdS_5 \times S^5$ on a line-defect (along the 3-direction), on which the endpoints of the strings appear as an electric charge density. The addition of angular momentum in the 12-plane to this probe D3/F1 system causes it to expand to a D5-tube with geometry $\mathbb{R}^4 \times \mathcal{C}$, so the array (1.2) is replaced by

$$\begin{array}{lccccccccc} \mathrm{D3}: & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ \\ \text{D5-tube}: & \bullet & \bullet & 3 & 4 & 5 & \underline{6} & \_ & \_ & \_ \;. \end{array} \tag{1.3}$$

In the second line the underlined direction indicates the orientation of the strings dissolved within the tube, whilst the remaining directions are those of the D3-branes also dissolved within the tube. The bullets indicate that the cross-section $\mathcal{C}$ of the D5-tube

[1] Although the expansion always leads to a closed cross-section because of charge conservation, 1/4-supersymmetry allows it to be an "open" curve without boundary (which must therefore extend to infinity).

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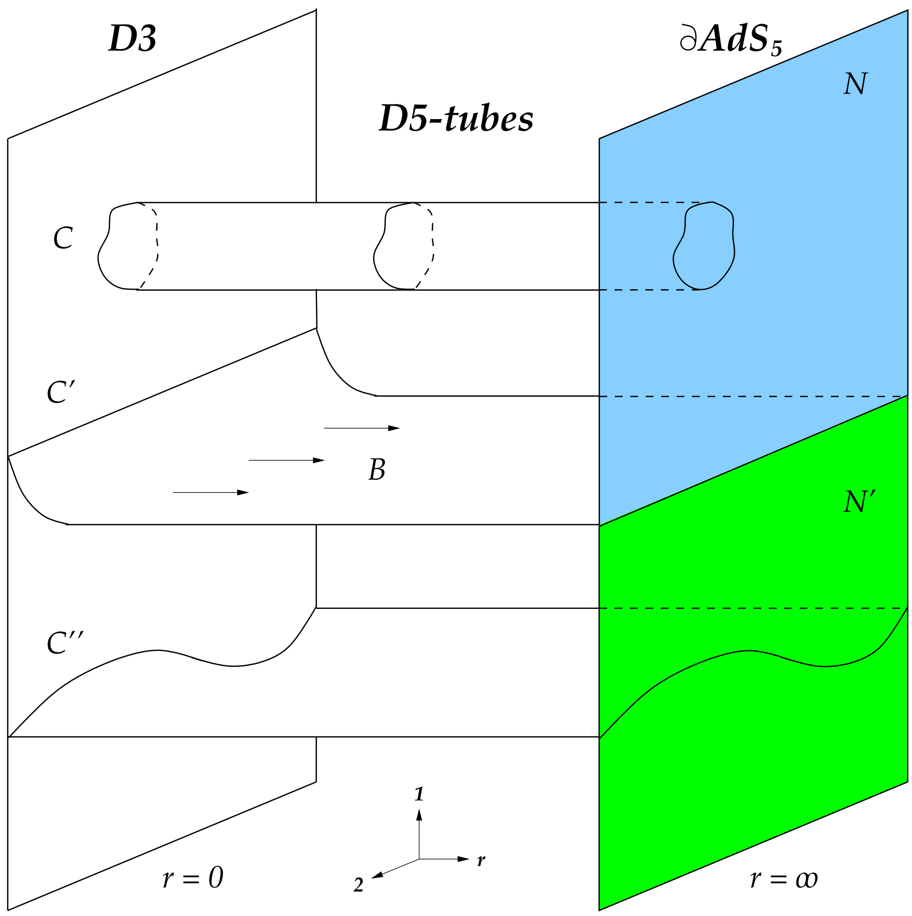

Figure 1: Here we have depicted three examples of D5-tubes intersecting D3-branes. (It should be understood that the D5-tubes continue through to the other side of the D3-branes.) The cross-sections of the tubes may be open ($\mathcal{C}'$ and $\mathcal{C}''$) or closed ($\mathcal{C}$). The numbers of D3-branes N and N' (here with $N \geq N'$) on either side of a D5-tube may differ in the case that the cross-section is a straight line ($\mathcal{C}'$). In this case the boundaries of the D3-branes deform the D5-tube and source a magnetic BI field B on it. If the D3-branes are replaced by an $AdS_5 \times S^5$ background then the D5-tubes intersect the boundary of AdS_5 (at $r \to \infty$) on a tubular defect with cross-section equal to that of the corresponding tube. The gauge group of the SYM theory is SU(N) on one side of the planar defect associated to the tube with linear cross-section $\mathcal{C}'$, and SU(N') on the other.

is restricted here to be a curve in the 12-plane (we will consider the general case in section 2). Since the D5-tube preserves the same supersymmetries as the collection of strings and D3-branes from which it originates, the intersection (1.3) is 1/8-supersymmetric. If the D3-branes are now replaced by the $AdS_5 \times S^5$ background then the resulting probe D5-tube ends at the AdS_5 boundary on a defect of geometry $\mathbb{R} \times \mathcal{C}$ (see figure 1). From the viewpoint of the $\mathcal{N} = 4$ SYM theory we conclude that the addition of electric charge and angular momentum to a line-defect causes it to expand to a defect of one dimension higher with geometry $\mathbb{R} \times \mathcal{C}$ while leaving unbroken four of the eight Poincaré supersymmetries preserved by the flat defect; conformal (super)symmetry is of course completely broken for non-linear $\mathcal{C}$. Analogous arguments show that a similar expansion occurs for the defects in columns I and II of table 1, with the exception of those involving D7-branes.

An $\mathbb{R}^q$-defect can thus be regarded as an expanded $\mathbb{R}^{q-1}$-defect in the limit in which $\mathcal{C}$ becomes a straight line; if in addition the electric and magnetic BI fields on the dual D-tube are set to zero then conformal (super)symmetry is restored and the resulting $\mathbb{R}^q$-defect reduces to one of the defects in table 1. Similarly, an $\mathbb{R}^{q-1}$-defect may be viewed as a collapsed q-dimensional defect in the limit in which $\mathcal{C}$ reduces to a point and the BI fields are set to zero. Thus all defects associated to the D3/Dp intersections in column I may be considered in a unified way;[2] the same applies to those defects in column II. We will see in section 5 how defects in columns I and II are related by considering configurations with multiple defects.

I	II
$(0\vert \mathrm{D3} \perp \mathrm{D1})$	$(0\vert \mathrm{D3} \perp \mathrm{D5})$
$(1\vert \mathrm{D3} \perp \mathrm{D3})$	$(1\vert \mathrm{D3} \perp \mathrm{D7})$
$(2\vert \mathrm{D3} \perp \mathrm{D5})$	
$(3\vert \mathrm{D3} \perp \mathrm{D7})$	

Table 1: 1/4-supersymmetric D3/Dp intersections.

The $\mathbb{R}^2$-defect is special because, being of codimension one, it can separate two *different* CFTs defined on either side of it; in particular, the ranks of their gauge groups may differ. In section 4 we investigate whether this is also possible for the non-planar $(\mathbb{R} \times \mathcal{C})$-defects uncovered here.

2. Anti-de Sitter supertubes

In this section we will explicitly verify that a D5-tube embedded in a D3-brane background with the orientation

$$\begin{array}{rccccccccc} \mathrm{D3}: & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ \\ \text{D5-tube}: & \bullet & \bullet & 3 & 4 & 5 & \underline{6} & \bullet & \bullet & \bullet \end{array} \tag{2.1}$$

indeed preserves a fraction of supersymmetry, as stated in the introduction; the calculation for any other Dp-tube is completely analogous. Note that (2.1) generalizes (1.3) in that the cross-section of the D5-tube is now a completely arbitrary curve in the $\mathbb{R}^5$-space corresponding to the 12789-directions; this is the D5-tube to which the D3/F1 system in (1.2) expands for a general angular momentum two-form in this $\mathbb{R}^5$-space.

The non-zero fields of the supergravity solution sourced by D3-branes take the form

$$\begin{aligned} ds^2 &= H^{-1/2} ds^2\left(\mathbb{E}^{(1,3)}\right) + H^{1/2} ds^2\left(\mathbb{E}^6\right) , \\ F_5 &= \omega + *\omega \,, \qquad \omega = \mathrm{vol}\left(\mathbb{E}^{(1,3)}\right) \wedge dH^{-1} \,, \qquad e^{\phi} = g_s \,, \end{aligned} \tag{2.2}$$

where H is a harmonic function on $\mathbb{E}^6$. If $H = 1$ this background reduces to flat space; if instead $H = 1 + R^4/r^4$, where $R^4 = g_s N$ and r is the radial coordinate in $\mathbb{E}^6$, then it describes N D3-branes at $r = 0$. The near-horizon limit is effectively obtained by removing the "1" in the harmonic function, in which case the geometry becomes $AdS_5 \times S^5$. For the moment we need not choose a specific form for H because most of the results we will obtain do not depend on it.

[2] The last entry in column I is not really associated to a defect because the intersection has codimension zero inside the D3-brane; rather, adding a probe D7-brane is associated to introducing dynamical "quarks" (or, more precisely, matter in the fundamental representation of the gauge group) in the gauge theory [10].

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In view of (2.1) we denote the coordinates in $\mathbb{E}^{(1,3)}$ as $\mathbf{x} = (x^0, x^3)$ and $\mathbf{X} = (X^a)$, where $a = 1, 2$, and for those in $\mathbb{E}^6$ we use $\mathbf{y} = (y^m)$, where $m = 4, 5, 6$, and $\mathbf{Y} = (Y^7, Y^8, Y^9)$. Since the D5-tube extends along the 03456-directions, we identify $\mathbf{x}$ and $\mathbf{y}$ with worldvolume coordinates. The additional worldvolume coordinate σ parametrizes the cross-section $\mathcal{C}$, which is specified by

$$\mathbf{X} = \mathbf{X}(\sigma)\,, \qquad \mathbf{Y} = \mathbf{Y}(\sigma)\,. \tag{2.3}$$

The BI field strength on the D5-tube takes the form

$$F = \mathcal{E}\, dx^0 \wedge dy^6 + \mathcal{B}\, dy^6 \wedge d\sigma\,. \tag{2.4}$$

The electric and magnetic components $\mathcal{E}$ and $\mathcal{B}$ correspond, respectively, to fundamental strings along the 6-direction and D3-branes along the 345-directions dissolved in the D5-brane. The Poynting momentum-density generated by the crossed electric and magnetic fields is responsible for the stability of the tube with an arbitrary cross-section [7, 8].

The supersymmetries preserved by a D5-brane are those generated by Killing spinors ϵ of the background that satisfy (see, for example, [11])

$$\Gamma\epsilon = \epsilon\,, \tag{2.5}$$

where Γ is the matrix appearing in the kappa-symmetry transformations of the D5-brane worldvolume fermions [12, 13]. For the case of interest here we have

$$\Delta\, \Gamma = \mathcal{B}\, \Gamma_{0345} I - \gamma \Gamma_{345}\left(\Gamma_{06} K - \mathcal{E}\right), \tag{2.6}$$

where $\{\Gamma_0, \dots, \Gamma_9\}$ are ten flat-space constant Dirac matrices, K and I are operators that act on SO$(1, 9)$ chiral complex spinors as

$$K\psi = \psi^*\,, \qquad I\psi = -i\psi\,, \tag{2.7}$$

and $\Delta = \sqrt{-\det(g + F)}$ is the DBI determinant, which in the present case is given by

$$\Delta = \sqrt{\mathcal{B}^2 + (1 - \mathcal{E}^2)(|\partial_\sigma \mathbf{X}|^2 + H|\partial_\sigma \mathbf{Y}|^2)}\,. \tag{2.8}$$

We have also introduced

$$\gamma = \sum_{i=1}^{2} \Gamma_i \partial_\sigma X^i + H^{1/2} \sum_{i=7}^{9} \Gamma_i \partial_\sigma Y^i\,. \tag{2.9}$$

For generic H the Killing spinors of the background (2.2) take the form

$$\epsilon = H^{-1/8} \epsilon_0\,, \tag{2.10}$$

where ϵ_0 is a constant spinor subject to the constraint

$$\Gamma_{0123} I \epsilon_0 = \epsilon_0 \tag{2.11}$$

associated to the presence of D3-branes along the 123-directions. This means that the background is invariant under sixteen real supersymmetries. This is enhanced to thirty-two supersymmetries in two particular cases. If $H = 1$ then the background is just the Minkowski vacuum and ϵ_0 is unconstrained. If $H = R^4/r^4$ then the background is $AdS_5 \times S^5$ and there are sixteen special conformal supersymmetries in addition to the Poincaré supersymmetries generated by ϵ_0. We will not need the specific form of the conformal supersymmetries in this section.

Equation (2.5) is satisfied for *arbitrary* cross-section if

$$|\mathcal{E}| = 1\,, \tag{2.12}$$

$\mathcal{B}$ is a constant-sign function on the D5-brane worldvolume, and ϵ_0 satisfies the additional constraints

$$\Gamma_{06} K \epsilon_0 = \mathrm{sgn}(\mathcal{E})\epsilon_0\,, \qquad \Gamma_{0345} I \epsilon_0 = \mathrm{sgn}(\mathcal{B})\epsilon_0\,. \tag{2.13}$$

These are respectively associated to the presence of string charge in the 6-direction and D3-brane charge in the 345-directions, as expected from the charges carried by the D5-tube. Since these constraints are compatible with each other and with (2.2), the D5-tube preserves four of the sixteen supersymmetries of the D3-brane background.[3] Note that, as anticipated, there is no projection corresponding to the presence of the D5-brane. Although supersymmetry allows $\mathcal{B}$ to depend on all the worldvolume coordinates, the Bianchi identity $dF = 0$ restricts it to depend only on y^6 and σ, and the equations of motion for F further disallow the dependence on y^6. Hence we conclude that $\mathcal{B}$ may be a constant-sign but otherwise arbitrary function of σ.[4] We would like to emphasize that if $\mathcal{B} \neq 0$ then $|\mathcal{E}| = 1$ is a *sub*critical electric field, by which we mean that the DBI lagrangian is real and nonzero. As anticipated, the BPS equations (that is, the conditions on $\mathcal{E}$ and $\mathcal{B}$) and the projections (2.13) we have derived for preservation of supersymmetry are independent of the form of H.

The supersymmetry preserved by the D5-tube is enhanced if the cross-section is a straight line in the 12-plane and $\mathcal{B}$ is constant: in this case the D5-tube preserves eight supersymmetries for any (subcritical) constant value of $\mathcal{E}$, because the matrix Γ becomes a constant matrix that commutes with that in (2.11).

3. Features of expanded defects

We will now analyze in more detail the features of the expanded defects in the $\mathcal{N} = 4$ SYM theory uncovered here, so we set $H = R^4/r^4$. Again we concentrate on the D5-tube for concreteness since the extension to other Dp-tubes is straightforward.

The probe D5-tube is embedded in $AdS_5 \times S^5$ in a way determined by the cross-section $\mathcal{C}$. The two particular cases in which $\mathcal{C}$ lies entirely along the 12-directions, which we refer to as "expansion along AdS_5", or along the 789-directions, which we refer to as "expansion along S^5", are especially interesting.

[3] If $H = 1$ the D5-tube preserves sixteen of the thirty-two Minkowski supercharges; the case of $AdS_5 \times S^5$ is discussed below.

[4] We have verified that all the remaining D5-brane equations of motion are satisfied under these circumstances.

3.1 Expansion along AdS_5

In this case $\mathbf{Y}$ is constant and the D5-tube intersects the boundary of AdS_5 on a defect of geometry $\mathbb{R} \times \mathcal{C}$, where $\mathcal{C}$ is a curve in the 12-plane. The induced metric on the D5-tube is

$$ds^2 = \frac{y^2 + Y^2}{R^2} \left(-dx_0^2 + d\sigma^2 + dx_3^2\right) + \frac{R^2}{y^2 + Y^2} dy^2 + \frac{R^2 y^2}{y^2 + Y^2} ds^2 \left(S^2\right), \tag{3.1}$$

where $y = |\mathbf{y}|$, $Y = |\mathbf{Y}|$ and we have set $|\partial_\sigma \mathbf{X}|^2 = 1$ without loss of generality by assuming an affine parametrization.

Let us first suppose that $Y = 0$. Although in this case the metric is that of $AdS_4 \times S^2$, this does *not* imply that part of the conformal isometries of AdS_5 are preserved, because the submanifold occupied by the brane is not mapped into itself by these isometries unless $\mathcal{C}$ is a straight line. From the viewpoint of the gauge theory it is clear that conformal symmetry (as well as translational and rotational symmetry in the 12-plane) must be broken if $\mathcal{C}$ is not a straight line since its size (or radius of curvature) introduces a scale. Similarly, conformal supersymmetry of $AdS_5 \times S^5$ is completely broken if the cross-section of the D5-tube is not a straight line, as we show in the Appendix. It follows that the corresponding tubular defect with generic $\mathcal{C}$ breaks all the conformal supersymmetries of the $\mathcal{N} = 4$ SYM theory; of course, it preserves four of the sixteen Poincaré supersymmetries, as we saw in the previous section.

It is thus clear that a necessary condition for preservation of conformal symmetry is that the cross-section of the D5-tube be a straight line, in which case the submanifold occupied by the D5-tube is invariant under the conformal isometries of AdS_5. However, this condition is not sufficient because the BI field strength on the worldvolume of the D5-tube is not invariant[5] unless $\mathcal{E}$ and $\mathcal{B}$ are set to zero (recall that precisely for a linear cross-section in the 12-plane this can be done while preserving Poincaré supersymmetry). We thus conclude that (part of) the conformal symmetries of AdS_5 are preserved by the D5-tube if and only if its cross-section is a straight line and $\mathcal{E} = \mathcal{B} = 0$; we show in the Appendix that the same applies for preservation of conformal supersymmetry. Under these circumstances the D5-tube reduces to an ordinary D5-brane, the intersection (2.1) becomes a $(2|\mathrm{D3} \perp \mathrm{D5})$ intersection, and the corresponding $\mathbb{R}^2$-defect preserves (part of) the conformal invariance of the $\mathcal{N} = 4$ SYM theory.

If $Y \neq 0$ then the metric (3.1) is that of $AdS_4 \times S^2$ for $y \gg Y$ but deviates from it in the interior of AdS_5. This means that the theory on the defect is not conformal even if $\mathcal{C}$ is a straight line and $\mathcal{E} = \mathcal{B} = 0$. In this case conformal invariance is broken because for non-zero Y the D3-D5 open strings, and hence the corresponding degrees of freedom on the defect, are massive.[6] The fact that the embedding becomes conformal for large y reflects the fact that these masses become negligible in the ultra-violet limit.

[5]Technically this means that its Lie derivative along the Killing vector fields that generate these isometries is non-zero.

[6]Note that this breaking of conformal invariance is explicit and supersymmetry-preserving, as opposed to the *non*-supersymmetry-preserving breaking by vacuum expectation values discussed in [6].

3.2 Expansion along S^5

Here we shall directly concentrate on the case in which $\mathcal{C}$ is a straight line. Since $\mathbf{X}$ is constant the induced metric on the D5-tube is

$$ds^2 = \frac{r^2}{R^2}\left(-dx_0^2 + dx_3^2\right) + \frac{R^2}{r^2}dr^2 + R^2\, ds^2\left(S^3\right). \tag{3.2}$$

Although the submanifold occupied by the D5-tube is indeed invariant under part of the conformal symmetries of AdS, as suggested by the $AdS_3 \times S^3$ form of the induced metric, these symmetries are again broken by the BI field strength. Note that setting the BI fields to zero is now incompatible with supersymmetry, since a pure D5-brane embedded as an $AdS_3 \times S^3$ submanifold is not supersymmetric[7] [6], as could be expected from the fact that it would seem to originate from a $(1|\mathrm{D3} \perp \mathrm{D5})$ intersection. It is precisely the non-zero worldvolume electromagnetic fields which make supersymmetry possible in this case. The dual defect is a supersymmetric non-conformal line-defect.

4. D3-branes ending on D5-tubes

The $\mathbb{R}^2$-defect dual to the $(2|\mathrm{D3} \perp \mathrm{D5})$ intersection permits the CFTs on either side of it to be different; the case of interest here is when the ranks of the gauge groups, N and N', differ. In the brane construction, this means that some D3-branes actually end on the D5-brane rather than intersect it, so that the difference in the numbers of D3-branes on either side is $\delta N \equiv N - N' \neq 0$. This imbalance has two effects on the D5-brane probe [1] (see figure 1). Firstly, it induces δN units of BI magnetic flux through the two-sphere in the 456-space, because the two-dimensional boundary of a D3-brane acts as a magnetic monopole charge in the five-dimensional worldspace of the D5-brane. Secondly, the D5-brane worldspace is deformed close to the boundaries of the D3-branes due to the imbalance in the forces exerted by their tensions.

Given the existence of supersymmetric (that is, stable) tubular defects with arbitrary cross-section, the question of whether the gauge group can also jump across these defects while preserving supersymmetry arises. To answer this we will look for the corresponding dual D5-tubes in $AdS_5 \times S^5$. We will argue that in the probe approximation used here these do not exist, and then discuss the meaning of this result and the limitations of the approximation.

The D5-tubes we seek should be oriented as indicated by (1.3), so we set $\mathbf{Y} = \mathbf{0}$, but the cross-section specified before by $\mathbf{X}(\sigma)$ should now be allowed to vary in the 456-directions to describe the deformation due to the D3-brane tension imbalance; in other words, we should now have $\mathbf{X} = \mathbf{X}(\sigma, \mathbf{y})$. We must also generalize the form of the BI field strength (2.4) to incorporate the magnetic flux sourced by the D3-brane boundaries. Therefore we must set

$$F = \mathcal{E}\, dx^0 \wedge dy^6 + \mathcal{B}\, dy^6 \wedge d\sigma + \frac{1}{2}\epsilon_{mnp} B^m\, dy^n \wedge dy^p \tag{4.1}$$

[7] Technically this manifests itself in the fact that the matrix Γ anti-commutes with that in (2.2) if the cross-section lies in the 789-space and $\mathcal{E} = \mathcal{B} = 0$.

and require

$$\delta N \equiv \frac{1}{2\pi}\int_{S^2} \vec{B}\cdot d\vec{S} \neq 0\,, \tag{4.2}$$

where $\vec{B} = (B^m)$ and S^2 is a two-sphere in the 456-space. Recall that $a, b, \ldots = 1, 2$ and $m, n, \ldots = 4, 5, 6$. Since we will need to distinguish between the 45-directions and the 6-direction because the symmetry between them is broken by $\mathcal{E}$ and $\mathcal{B}$, we will further set $\mathbf{y} = (y^m) = (y^i, y^6)$ with $i, j = 4, 5$. Note that in principle we should expect $\mathcal{B}$ and B^m to depend on σ and y^m, of course subject to the conditions

$$\begin{aligned} \partial_m B^m &= 0\,, && (4.3)\\ \partial_\sigma B_4 &= \partial_5 \mathcal{B}\,, && (4.4)\\ \partial_\sigma B_5 &= -\partial_4 \mathcal{B}\,, && (4.5)\\ \partial_\sigma B_6 &= 0 && (4.6) \end{aligned}$$

implied by the Bianchi identity $dF = 0$.

Under these circumstances the kappa-symmetry matrix takes the form

$$\begin{aligned} \Delta\Gamma = {} & \mathcal{B}\,\Gamma_{0345} I + H^{-1} B^m\, \partial_{[m} X^1 \partial_{\sigma]} X^2\, \Gamma_{0123} I \\ & + \left[\Gamma_{345}\left(\gamma + H^{-1/2}\partial_{[i} X^1 \partial_{\sigma]} X^2 \Gamma_{i12}\right) I + H^{-1/2} B_6 \Gamma_3\, \gamma\, K I\right] (\Gamma_{06} K - \mathcal{E}) \\ & + \Gamma_{03} K I\, (\mathcal{B} B_6 + \Gamma_{1245} \partial_{[6} X^1 \partial_{\sigma]} X^2) + H^{-1/2}\left[\mathcal{B}\partial_i X^a \Gamma_{ai}\Gamma_{0345} I + B_i \partial_\sigma X^{[a}\Gamma^{b]}\Gamma_i \Gamma_{0123} I\right] \\ & + H^{-1}\left[\frac{1}{6}\Gamma_{0123}\gamma K I\, \Gamma_{[m}\partial_n X^1 \partial_{p]} X^2 + \Gamma_{0123} I \left(\mathcal{B} - \frac{1}{3}\mathcal{E}\Gamma_0\,\gamma\right) \partial_{[4} X^1 \partial_{5]} X^2 \right. \\ & \qquad \left. + \frac{1}{3}\Gamma_{03a} K I\, \delta_{bc}\Gamma_{[m}\partial_n X^a \partial_{p]} X^b \partial_\sigma X^c - \frac{1}{3}\mathcal{E}\Gamma_{3a} I\, \delta_{bc} \partial_{[i} X^a \partial_{j]} X^b \partial_\sigma X^c\right], \end{aligned} \tag{4.7}$$

where

$$\gamma = \partial_\sigma X^a \Gamma_a\,. \tag{4.8}$$

It follows that (2.5) is satisfied if we impose the projections (2.11) and (2.13), as well as the BPS conditions (2.12) and

$$\begin{aligned} B_i \partial_\sigma X^a &= -\mathcal{B}\,\epsilon^{ab}\partial_i X^b\,, && (4.9)\\ \mathcal{B}\, B_6 &= \partial_{[6} X^1 \partial_{\sigma]} X^2\,, && (4.10)\\ 0 &= \partial_{[m} X^1 \partial_{n]} X^2\,, && (4.11) \end{aligned}$$

in which case the DBI determinant reduces to

$$\Delta = \left|\mathcal{B} + H^{-1} B^m\, \partial_{[m} X^1 \partial_{\sigma]} X^2\right|. \tag{4.12}$$

Note the non-linearity of the BPS equations, as well as the asymmetry between the 45- and the 6-directions.

The projections above are expected given the charges carried by the D5-tube. In particular, we need to impose condition (2.11) even if $H = 1$, that is, even if there are no background D3-branes, because these should arise here as a "spike" excitation on the D5-tube described by $\mathbf{X}(\sigma, \mathbf{y})$.

It is well-known that the rank of the gauge group may change while preserving supersymmetry across an $\mathbb{R}^2$-defect on which $\mathcal{E} = \mathcal{B} = 0$ [1]. Let us therefore first show that this is also posssible with non-zero BI fields. By rotational invariance in the 12-plane, we assume without loss of generality that the linear cross-section is aligned with the 2-axis, that is, we set

$$X^2 \equiv \sigma\,, \qquad X^1 \equiv X(\mathbf{y})\,. \tag{4.13}$$

The BPS equations then simplify to

$$B_i = \mathcal{B}\,\partial_i X\,, \qquad \mathcal{B}\,B_6 = \partial_6 X\,. \tag{4.14}$$

Under these circumstances the equations of motion for the BI gauge field on the D5-brane in the background (2.2) reduce to the conditions that $\mathcal{B}$ is constant and that

$$\partial_4^2 X + \partial_5^2 X + \mathcal{B}^{-2}\partial_6^2 X = 0\,. \tag{4.15}$$

The latter equation coincides with the divergence-free condition (4.3) for B^m upon using (4.14); since $\mathcal{B}$ is constant, it is solved by any harmonic function of $\tilde{\mathbf{y}}$, where $\tilde{y}^i = y^i$ and $\tilde{y}^6 = \mathcal{B} y^6$. The choice $X = \delta N/4\pi|\tilde{\mathbf{y}}|$ satisfies (4.2), and represents δN open D3-branes ending on the D5-tube at $\mathbf{y} = \mathbf{0}$.

We now turn to non-linear cross-sections. We will show that the cross-section cannot be non-linear unless $\delta N = 0$. It will suffice to prove this for a circular cross-section since any curve is approximated locally by a circle, and the condition for preservation of supersymmetry (2.5) is local. Let us therefore set

$$X^1 = R(\mathbf{y})\sin\sigma\,, \qquad X^2 = R(\mathbf{y})\cos\sigma\,. \tag{4.16}$$

The BPS eqs. (4.9)–(4.11) then reduce to

$$R\,B_i = \mathcal{B}\,\partial_i R\,, \qquad \mathcal{B}\,B_6 = R\,\partial_6 R\,. \tag{4.17}$$

Since R is σ-independent, these equations together with (4.4)–(4.6) imply that B^m cannot depend on σ, and also that $\mathcal{B}$ may only depend on y^6. The D5-brane equations of motion then force R to take the form

$$R(\mathbf{y}) = f(y^4, y^5)\,|\mathcal{B}(y^6)|^{1/2}\,, \tag{4.18}$$

where f and $\mathcal{B}$ must satisfy

$$\left(\partial_4^2 + \partial_5^2\right)\log f = \lambda\, f^2\,, \qquad \partial_6^2 \log\mathcal{B} = -2\lambda\,\mathcal{B}\,, \tag{4.19}$$

for some undetermined constant λ. We seek a solution for which R tends to a non-zero constant far away from the D3-brane boundaries, which is at $y \to \infty$ if we assume (without loss of generality) that the D3-branes end on the D5-tube at $y = 0$. Since this means that the left-hand sides of (4.19) vanish asymptotically, we must set $\lambda = 0$. If we additionally restrict ourselves to solutions invariant under rotations in the 45-plane, then

$$f = a\left(y_4^2 + y_5^2\right)^b, \qquad \mathcal{B} = c\, e^{dy^6}, \tag{4.20}$$

for constants a, b, c, d. The requirement that R tends to a *non-zero* constant implies that $c = d = 0$, in which case R is everywhere constant and $B^m = 0$. This is therefore a particular case of the general D5-tube with an arbitrary but $\mathbf{y}$-independent cross-section and $\delta N = 0$.

Although the above result was derived for a particular ansatz for the BI fields (4.1), this ansatz is grounded in a physical understanding of supertubes, and hence we consider it unlikely that there exists a supersymmetric solution not captured by this ansatz. Another caveat is that effects beyond the probe approximation could allow a supersymmetric non-linear cross-section provided that its radius of curvature is of the order $N/\delta N$, consistent with the fact that in the limit $\delta N/N \to 0$ only linear cross-sections are supersymmetric.

5. Multiple defects

In this section we discuss configurations with multiple defects, some of which involve defects from both columns I and II in table 1.

A first observation is that any number of defects with geometries $\mathbb{R}^q \times \mathcal{C}_i$ (for fixed q), where the $\mathbb{R}^q$-factors are mutually-aligned and the cross-sections $\mathcal{C}_i$ are arbitrary, preserve 1/4 of the Poincaré supersymmetries of the $\mathcal{N} = 4$ SYM theory. (Three of them with $q = 1$ have been depicted in figure 1.) This follows immediately from the fact that Dp-tubes with mutually-aligned axes and arbitrary cross-sections preserve 1/4-supersymmetry, as was shown in [8] by exhibiting a 1/4-supersymmetric supergravity solution representing such a multi-tube configuration.[8] Note that this includes as a particular case parallel defect/anti-defect configurations, for which the dual system consists of a supersymmetric parallel brane/anti-brane pair [8, 16].

We have shown that the line-defect of column I can be viewed as a collapsed tubular two-dimensional defect. This suggests that the line-defect could have an endpoint on a planar $\mathbb{R}^2$-defect, or that it could even be smoothly attached to it. The dual configuration in $AdS_5 \times S^5$ that realizes this possibility is described by the array

$$\begin{array}{llllllllll} \text{D3:} & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ \\ \text{D5:} & \_ & 2 & 3 & 4 & 5 & 6 & \_ & \_ & \_ \\ \text{D3:} & 1 & \_ & \_ & 4 & 5 & \_ & \_ & \_ & \_ \end{array} . \tag{5.1}$$

As usual the D3-branes at the top provide the $AdS_5 \times S^5$ background. The probe D3-brane at the bottom ends on the probe D5-brane, that is, it extends only along the (say) positive values of the 1-coordinate. From the viewpoint of the SYM theory this represents a line-defect ending on an $\mathbb{R}^2$-defect. Since the D3-brane can be viewed as a smooth solitonic

[8]For circular cross-sections the supergravity solution was first found in [14], and solutions in Matrix theory were constructed in [15].

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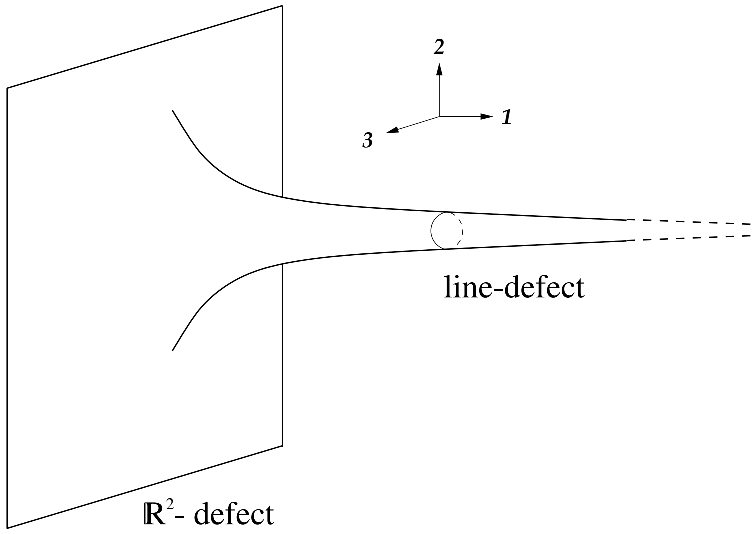

Figure 2: A line-defect ending on an $\mathbb{R}^2$-defect.

excitation of the D5-brane [17, 18], one might expect that the line defect can be similarly viewed as a solitonic excitation of the conformal field theory on the $\mathbb{R}^2$-defect as depicted in figure 2.

The CFT on the $\mathbb{R}^2$-defect associated to the $(2|\mathrm{D3} \perp \mathrm{D5})$ intersection was deduced in [5, 19], following the earlier work of [3, 4]. Essentially, it is a linear $\mathcal{N} = 4$ $d = 3$ sigma-model with $4N$ scalar fields but with additional couplings to the pullbacks of the $d = 4$ SYM fields.[9] A vortex solution of this theory could interpolate between a region of non-zero scalar fields at the vortex core to an asymptotic region of vanishing scalar fields far away from the core. Due to the coupling to the four-dimensional background fields, non-vanishing scalar fields on the defect imply discontinuities of the Higgs fields across the defect. To take this properly into account would require going beyond the probe approximation.

Given that a line-defect can end on a planar defect, one may wonder whether a tubular defect formed by expansion of a line-defect can also end on a planar defect. This would seem to require a corresponding bulk configuration in which a D5-tube has a boundary on a D5-brane, and there is no known supersymmetric configuration of this kind; a possible resolution is that the D5-brane expands *locally* to a D7-tube, on which the D5-tube can end. This illustrates a rather general problem in any attempt to find a supertube *ending* on another brane, as opposed to merely intersecting it. An examination of all possibilities [20] shows that the only case in which the problem does not arise is that recently discussed [9] of a D2-tube ending on a D4-brane, and the T-dual cases in which a Dp-tube ends on a D$(p+2)$-brane. Note that a naive application of T-duality rules to systems with branes

[9] The same field theory, but in a lower dimension, should therefore give the dynamics of the line- and point-defects associated to the $(1|\mathrm{D3} \perp \mathrm{D3})$ and $(0|\mathrm{D3} \perp \mathrm{D1})$ intersections because each of them can be regarded as a collapsed $\mathbb{R}^2$-defect.

Figure 3: On the left-hand side a line-defect ends on a point-defect; on the right-hand side the expanded line-defect ends on the expanded point-defect after the addition of angular momentum.

ending on branes can lead to incorrect results. For example, T-duality along a direction longitudinal to the D4-brane and orthogonal to the D2-brane that ends on it might seem to transform this system into one in which a D3-brane has a one-dimensional boundary (on another D3-brane), which is impossible.

The line-defect of column I in table 1 can also end on the point-defect associated to the $(0|\mathrm{D3} \perp \mathrm{D5})$ intersection of column II (see figure 3). The corresponding brane configuration is

$$\begin{array}{lccccccccc}
\mathrm{D3:} & 1 & 2 & 3 & _ & _ & _ & _ & _ & _ \\
\mathrm{D5:} & _ & _ & _ & 4 & 5 & 6 & 7 & 8 & _ \\
\mathrm{D3:} & _ & _ & 3 & 4 & 5 & _ & _ & _ & _ \quad ,
\end{array} \tag{5.2}$$

where the probe D3-brane at the bottom ends on the probe D5-brane. If strings along the 9-direction and angular momentum in the 12-plane are added, then both probes can expand supersymmetrically yielding a configuration in which a D5-tube ends on a D7-tube:

$$\begin{array}{rccccccccc}
\mathrm{D3:} & 1 & 2 & 3 & _ & _ & _ & _ & _ & _ \\
\mathrm{D7\text{-}tube:} & \bullet & \bullet & _ & 4 & 5 & 6 & 7 & 8 & \underline{9} \\
\mathrm{D5\text{-}tube:} & \bullet & \bullet & 3 & 4 & 5 & _ & _ & _ & \underline{9} \quad ,
\end{array} \tag{5.3}$$

Both tubes share a common cross-section $\mathcal{C}$ in the 12-plane. From the viewpoint of the CFT theory, a tubular $\mathbb{R} \times \mathcal{C}$-defect with axis along the 3-direction ends on a one-dimensional $\mathcal{C}$-defect (see figure 3). In the limit in which $\mathcal{C}$ becomes a straight line, the $\mathcal{C}$-defect reduces to the line-defect associated to the second intersection in column II of table 1, and the configuration above becomes an $\mathbb{R}^2$-defect with a boundary line-defect. For a line in the (say) 2-direction this corresponds to the array

$$\begin{array}{lccccccccc}
\mathrm{D3:} & 1 & 2 & 3 & _ & _ & _ & _ & _ & _ \\
\mathrm{D7:} & _ & 2 & _ & 4 & 5 & 6 & 7 & 8 & 9 \\
\mathrm{D5:} & _ & 2 & 3 & 4 & 5 & _ & _ & _ & 9 \quad .
\end{array} \tag{5.4}$$

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A. Special conformal supersymmetries

If the cross-section of the D5-tube is a line in the 12-plane (at $\mathbf{Y} = 0$) and $\mathcal{E} = \mathcal{B} = 0$ then eight of the sixteen special conformal supersymmetries of $AdS_5 \times S^5$ are preserved (together with eight of the sixteen Poincaré supersymmetries) [6]. We show here that these conditions are not only sufficient but also necessary for preservation of conformal supersymmetry, as stated in section 3.1.

It is convenient to set $R = 1$ and to use spherical coordinates $\{r, \theta_\alpha\}$ ($\alpha = 1, \ldots, 5$) for the $\mathbb{R}^6$-space transverse to the D3-branes such that $\mathbf{Y} = 0$ corresponds to $\theta_1 = \theta_2 = \theta_3 = \pi/2$. The D5-tube then wraps the S^2 within the S^5 parametrized by θ_4, θ_5, and its embedding in AdS_5 is specified by $\mathbf{X}(\sigma)$. In these coordinates the BI field strength (2.4) and the DBI determinant take the form

$$F = \mathcal{E}\cos\theta_4 dx_0 \wedge dr - \mathcal{E} r \sin\theta_4 dx_0 \wedge d\theta_4 + \mathcal{B}\cos\theta_4 dr \wedge d\sigma - \mathcal{B} r \sin\theta_4 d\theta_4 \wedge d\sigma\,, \tag{A.1}$$

$$\Delta = r^2 \sin\theta_4 \sqrt{\mathcal{B}^2 + (1-\mathcal{E}^2)|\partial_\sigma \mathbf{X}|^2}\,, \tag{A.2}$$

whereas the kappa-symmetry matrix is

$$\begin{aligned}\Delta\Gamma = r^2 \sin\theta_4 \Gamma_{3\theta_5} I \,\{&[\Gamma_{0r\theta_4} K + \mathcal{E}\left(\cos\theta_4 \Gamma_{\theta_4} + \sin\theta_4 \Gamma_r\right)]\gamma - \\ &- \mathcal{B}\Gamma_0 \left(\cos\theta_4 \Gamma_{\theta_4} + \sin\theta_4 \Gamma_r\right)\}\,.\end{aligned} \tag{A.3}$$

Here γ is as defined in (4.8) and $\{\Gamma_0, \ldots \Gamma_3, \Gamma_r, \Gamma_{\theta_1}, \ldots \Gamma_{\theta_5}\}$ are ten tangent-space constant Dirac matrices.

Following [6] closely we write the Killing spinors for $AdS_5 \times S^5$ as

$$\epsilon = r^{1/2} h(\theta_\alpha)\eta_1 + \left[-r^{-1/2}\Gamma_r + r^{1/2}(x_0\Gamma_0 + X^a\Gamma_a + x_3\Gamma_3)\right] h(\theta_\alpha)\eta_2\,, \tag{A.4}$$

where η_1 and η_2 are *constant* sixteen-component complex spinors of negative and positive chirality, respectively, subject to the additional constraints

$$\Gamma_{0123} I\, \eta_1 = \eta_1\,, \qquad \Gamma_{0123} I\, \eta_2 = -\eta_2\,, \tag{A.5}$$

and the matrix

$$h(\theta_\alpha) = \exp\left(\frac{\theta_1}{2}\Gamma_{r\theta_1}\right)\exp\left(\frac{\theta_2}{2}\Gamma_{\theta_1\theta_2}\right)\times \tag{A.6}$$

$$\times \exp\left(\frac{\theta_3}{2}\Gamma_{\theta_2\theta_3}\right)\exp\left(\frac{\theta_4}{2}\Gamma_{\theta_3\theta_4}\right)\exp\left(\frac{\theta_5}{2}\Gamma_{\theta_4\theta_5}\right) \tag{A.7}$$

accounts for the tangent-space rotation induced by the change from Cartesian to spherical coordinates in $\mathbb{R}^6$. If $\eta_2 = 0$ then the remaining eight independent complex Killing spinors are the Poincaré spinors (2.10). The Killing spinors ϵ with $\eta_2 \neq 0$ (but η_1 not necessarily zero) correspond to special conformal supersymmetries, as indicated by their dependence on the coordinates along the D3-branes.

Since (2.5) must hold at all points of the D5-tube, terms with different functional dependencies on the worldvolume coordinates must vanish independently. The term with $r^{3/2}$ dependence in (2.5) reduces to

$$\begin{aligned}&\Gamma_{3\theta_5} I\,\{[-\Gamma_{0r\theta_4}K + \mathcal{E}(\cos\theta_4\Gamma_{\theta_4} - \sin\theta_4\,\Gamma_r)]\gamma - \\ &\quad - \mathcal{B}\Gamma_0(\cos\theta_4\Gamma_{\theta_4} - \sin\theta_4\,\Gamma_r)\}\, h(\theta_\alpha)\eta_2 = \sqrt{\mathcal{B}^2 + (1-\mathcal{E}^2)|\partial_\sigma\mathbf{X}|^2} h(\theta_\alpha)\eta_2\,.\end{aligned} \tag{A.8}$$

The term with x_0 dependence similarly reduces to

$$\Gamma_{3\theta_5} I \{[-\Gamma_{0r\theta_4} K + \mathcal{E}(\cos\theta_4 \Gamma_{\theta_4} + \sin\theta_4 \Gamma_r)]\gamma + \\ + \mathcal{B}\Gamma_0(\cos\theta_4\Gamma_{\theta_4} + \sin\theta_4\Gamma_r)\} h(\theta_\alpha)\eta_2 = \sqrt{\mathcal{B}^2 + (1-\mathcal{E}^2)|\partial_\sigma \mathbf{X}|^2} h(\theta_\alpha)\eta_2 \,. \quad \text{(A.9)}$$

Subtracting the two equations above, we see that

$$[\mathcal{E}\sin\theta_4\,\Gamma_r\gamma + \mathcal{B}\cos\theta_4\,\Gamma_{0\theta_4}]\, h(\theta_\alpha)\eta_2 = 0\,. \quad \text{(A.10)}$$

Using the commutation properties of the gamma matrices to bring $h(\theta_\alpha)$ through to the left, we find that

$$\left[\exp\left(\frac{\theta_5}{2}\Gamma_{\theta_4\theta_5}\right)\Gamma_{\theta_1}\gamma\sin\theta_4\mathcal{E} - \exp\left(-\frac{\theta_5}{2}\Gamma_{\theta_4\theta_5}\right)\Gamma_{0\theta_4}\cos\theta_4\mathcal{B}\right]\eta_2 = 0\,. \quad \text{(A.11)}$$

Since the two terms must vanish independently for all θ_4, θ_5 and the matrices in each term are invertible, we see that this equation can be satisfied with non-zero η_2 only if $\mathcal{E} = \mathcal{B} = 0$. If we substitute this condition back into (A.9) we find that

$$\Gamma_{03r\theta_4\theta_5} K I \frac{\gamma}{|\partial_\sigma \mathbf{X}|}\tilde{\eta} = \tilde{\eta}\,, \quad \text{(A.12)}$$

where $\tilde{\eta} = h(\theta_\alpha)\eta_2$. Fixing θ_α and demanding that this equation holds for all σ for some non-zero η_2 (note that $h(\theta_\alpha)$ is invertible) requires that the matrix $\gamma/|\partial_\sigma \mathbf{X}|$ is constant, which in turn implies that $\partial_\sigma X^a$ are constants, that is, that the cross-section is a straight line.

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